Removing Some Dissonance from the Social Discount Rate Debate

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Abstract

In an economy with a capital income tax distortion, the social discount rate (SDR) should reflect the social opportunity cost of capital rather than the social rate of time preference (consumption rate of interest) to ensure that public investments can produce Pareto improvements. The marginal cost of funds may exceed unity for a lump sum tax, but it is irrelevant for project evaluation. Even if a social welfare improvement is judged to be possible without passing the compensation test, the SDR should still reflect the social opportunity cost of capital to ensure that the project is the most efficient use of public funds.

1 Introduction

This paper is an attempt to reconcile three criteria for project evaluation that are prominent in the literature: the social opportunity cost of capital (SOC) criterion proposed by Harberger (1973) and Sandmo-Dreze (1971) (HSD), which discounts benefits and costs at a rate that reflects the social opportunity cost of capital (a weighted average of the pre-tax and after-tax rates of return); the social rate of time preference approach (STP) proposed by Marglin (1964), Feldstein (1972), Bradford (1975) and Lind (1982), which converts benefits and costs into their "consumption equivalents" by shadow pricing all investment displaced or induced and discounting at the social rate of time preference (typically represented by the after-tax rate of return); and the marginal cost of funds (MCF) approach recently proposed by Liu (2003) and Liu et al (2004), which discounts benefits at either the after-tax rate of return or the pre-tax rate of return depending upon whether they are intra-generational or inter-generational, and discounts costs (including any "indirect revenue effects") at the pre-tax rate of return, but multiplies all costs and indirect revenue effects by the MCF.

The MCF criterion recognizes that if there are pre-existing distortions, in particular a capital income tax distortion, raising a dollar of revenue via a lump sum tax will cost more than a dollar because the tax increase will reduce saving and lower capital income tax revenue. Liu (2003) argues that the MCF must be an integral part of any multi-period project evaluation, and the SOC and STP criteria are both deficient because they fail to take the MCF into account. However, I believe there is some misunderstanding about how to apply the SOC and STP criteria. I show that if these criteria are implemented properly they are both consistent with the MCF criterion; all criteria correctly identify welfare improving projects. This three way equivalence is fragile however; it holds only in the context of an infinitely lived representative agent model. Using an overlapping generations model in which individuals have finite lives and make no bequests I show that the SOC criterion and the MCF criterion both correctly identify Pareto improving projects, but the STP criterion fails to meet this standard.

The SOC and STP criteria take the distortionary tax structure as given, so the marginal dollar of revenue to finance any project is obtained via lump sum taxation (or government borrowing with the debt serviced via lump sum taxation). Liu (2003) claims that when there are pre-existing distortions even a lump sum tax can carry an excess burden. However, whether a lump sum tax carries an excess burden is a matter of semantics.[1] A lump sum tax may be the least damaging form of tax, in which case any other form of tax would carry an excess burden. On the other hand, when distortionary taxes are present a lump sum tax may raise less revenue than the tax itself, in which case the damage exceeds the amount of tax revenue raised. For purposes of multi-period project evaluation, if the project produces benefits that are treated as income a straightforward application of the SOC criterion proposed by HSD is appropriate. For

[1] See e.g. Ballard and Fullerton (1992).

any project whose benefits are not treated as income there will be "indirect revenue effects", but they can be incorporated by adding to or subtracting from the project's net benefits. In either case, it is not necessary to take the MCF into account in applying the SOC criterion. On the other hand, the MCF criterion is also perfectly valid, but the marginal cost of funds is sensitive to the model and so is the appropriate discount rate for evaluating project benefits. This makes the MCF criterion difficult to implement compared to the SOC criterion.

Insofar as the STP criterion is concerned, it turns out to be equivalent to the MCF criterion in the context of an infinitely lived representative agent model provided that the proportions of project expenditure that are drawn from consumption and investment are properly specified. But this equivalence breaks down in an overlapping generations model in which benefits and costs accrue to different generations.

The points raised here might seem somewhat arcane and esoteric, but they turn out to be extremely important in practical applications of social cost-benefit analysis. For over 40 years the economics profession has debated what is the appropriate social discount rate. The wide disparity in preferred numerical values for the social discount rate among the experts that is reported in Weitzman (2001) reflects either fundamental disagreement about what are the appropriate conceptual foundations for the social discount rate or serious misunderstanding about what social cost-benefit analysis is supposed to achieve. Those who propose to discount benefits and costs at the relatively low social rate of time preference (approximated by the after tax rate of return) seem to believe that concerns about opportunity cost can be adequately addressed by shadow pricing all investment displaced or induced by the project. It is debatable whether they fully appreciate that the procedure implicitly adopts a particular utilitarian social welfare function that not everyone would accept, and in addition ignores more efficient ways of achieving the income distribution that the project achieves. It is the aim of this paper to shed further light on these issues and in the process to remove some dissonance from the social discount rate debate.

2 Infinitely Lived Representative Agent Model

Consider the following simplified version of the infinitely lived representative agent model used by Liu (2003). Assume the representative agent earns an exogenous pre-tax wage, w, and pre-tax rate of return on assets, ρ, but incurs a tax, T, on labour income and a tax at proportional rate τ on capital income . There are two goods available in each period: a composite private good c^t, and a publicly provided good g^t. Labour supply is exogenous, so any tax on labour income is effectively a lump sum tax.[2]

Given the time path of the publicly provided good, the representative agent chooses a time path for private consumption $\{c^t\}$ to maximize

$\Sigma_{t=0}^{\infty}\beta^t U(c^t, g^t)$

[2] The model could be generalized by introducing labour-leisure choice as in Liu (2003), but this would just complicate the algebra without changing the basic insights.

subject to
$\Sigma_{t=0}^{\infty} c^t/(1+r)^t = \Sigma_{t=0}^{\infty}(w - T^t)/(1+r)^t + A_0$

Here g^t is the supply of the publicly provided good in period t, A_0 is initial assets, and T^t is the lump sum tax in period t. The consumer's discount rate is the after tax rate of return $r = (1-\tau)\rho$.

From the first order conditions, consumption in each period is a function of the present value of lump sum taxes and the time path of the publicly provided good so $c^t(g, T)$. Well-being can therefore be written as $U(c(g, T), g) = V(g, T)$.

A project is represented by a sequence of expenditures $\{dI^t\}$ and outputs $\{dg^t\}$. At issue is whether the project is worthwhile.

Following Liu (2003), I assume that the government cannot appropriate the benefits of the publicly provided good through user fees or normal market transactions. The government's budget constraint therefore requires that the discounted sum of tax revenue $\{R^t\}$ minus project expenditures $\{I^t\}$ must be equal to its initial net indebtedness D^0. The discount rate is the pre-tax rate of return ρ. Thus,
$\Sigma_{t=0}^{\infty}(R^t - I^t)/(1+\rho)^t = D^0$

Tax revenue in period t is lump sum taxes plus capital income taxes, and capital income taxes depend upon assets in period t, where assets in period t depend upon the present value of the time stream of lump sum taxes and conceivably as well the time stream of the publicly provided good.
$R^t = T^t + \tau\rho A^t(g, T)$

Now consider a small project that requires an initial expenditure dI^0 and produces a stream of output dg^t in periods $t = 1$ and thereafter. The project is worthwhile if the representative agent is made better off. Since $V(g, T)$ is her indirect utility function and assuming, with no loss in generality, that the project is financed by an increase in lump sum taxes in period 0 [3], the project will make the representative agent better off if
$\Sigma_{t=1}^{\infty}(\partial V/\partial g^t)dg^t + (\partial V/\partial T_0)dT_0 > 0$

Dividing through by $\partial V/\partial T_0$ and making use of the envelope theorem, this can be re-written as
$\Sigma_{t=1}^{\infty}\frac{\partial U/\partial g^t}{\partial U/\partial c^0}dg^t - dT_0 > 0$

The project's benefit in period t, denoted by B^t, is valued at $p_g^t dg^t$, where $p_g^t = (\partial U/\partial g^t)/(\partial U/\partial c^t)$ represents the marginal rate of substitution between the publicly provided good and the composite private good in period t. From the first order conditions for a consumer optimum $\beta^t(\partial U/\partial c^t)(1+r)^t = \partial U/\partial c^0$. Therefore the representative agent will be better off with the project if the benefits discounted at the after tax rate of return exceed the lump sum tax increase required to finance the project, i.e. if

(1) $\Sigma_{t=1}^{\infty} B^t/(1+r)^t - dT_0 > 0.$

A project costing dI^0 that is financed by a lump sum tax increase dT^0 is fiscally feasible (i.e. satisfies the government budget constraint) if the present

[3] Ricardian equivalence holds in the ILA model, so the timing of any lump sum tax increase is irrelevant.

value of the additional tax revenue collected is equal to the project's expenditure requirements. Thus

(2) $dT^0 + \Sigma_{t=1}^{\infty} dR^t/(1+\rho)^t = dI^0$

The second term on the left hand side captures the induced effect of the project and its financing on the present value of capital income tax revenue. This depends on the impact of the lump sum tax increase on assets in all subsequent periods as well as any impact of the project itself on assets. Since $R^t = T^t + \tau\rho A^t$, the change in tax revenue in periods $t = 1$ and thereafter is

$dR^t = \tau\rho dA^t = \tau\rho \left[\Sigma_{i=1}^{\infty}(\partial A^t/\partial g^i)dg^i + (\partial A^t/\partial T^0)dT^0\right]$

Define $IR^t = \tau\rho\Sigma_{i=1}^{\infty}(\partial A^t/\partial g^i)dg^i$ as the "indirect revenue effect" of the project in period t. It represents the impact of the project on capital income tax revenue in period t.

For simplicity, I will focus on a project that produces a constant stream of output in perpetuity beginning in period 1 so $dg^t = dg$ for $t = 1, ...\infty$. In addition, I will assume that the representative agent's pure rate of time preference is equal to the after tax rate of return so she consumes the annuity value of wealth. The project therefore produces a perpetual stream of benefits worth B beginning in period 1. In addition, private consumption in each period equals permanent income so $c^t = w - T + rA$. A lump sum tax increase of dT^0 will therefore reduce consumption in periods 0 and thereafter by $dc^t = -rdT^0/(1+r)$, assets in periods 1 and thereafter will decrease by $dA^t = -dT^0/(1+r)$, and tax revenue in periods $t = 1$ and thereafter will decrease by $dR^t = IR^t - \tau\rho dT^0/(1+r)$.

Now substitute for dR^t in equation (2) and we find that a project is fiscally feasible if

(3) $dT^0 \left[1 - \tau/(1+r)\right] + \Sigma_{t=1}^{\infty} IR^t/(1+\rho)^t = dI^0$

The expression in square brackets can be interpreted as the "marginal cost of funds" (MCF) for a lump sum tax increase. The reason is as follows. The MCF is the ratio of the increase in lump sum taxes to the increase in the present value of total taxes collected. Thus

$MCF = 1/\left[\Sigma_{t=0}^{\infty}(dR^t/dT^0)/(1+\rho)^t\right] = 1/\left[1 + \Sigma_{t=1}^{\infty}\tau\rho(dA^t/dT^0)/(1+\rho)^t\right]$

Since $dA^t/dT^0 = -1/(1+r)$, the MCF simplifies to $MCF = \rho(1+r)/r(1+\rho)$.

Now use equation (3) to eliminate dT^0 from equation (1) and we find that the representative agent will be better off with the project provided that

(4) $\Sigma_{t=1}^{\infty} B^t/(1+r)^t - MCF\left[dI^0 - \Sigma_{t=1}^{\infty} IR^t/(1+\rho)^t\right] \geq 0$

According to equation (4), the project is worthwhile provided that the benefits discounted at the after tax rate of return exceed the project's direct expenditures minus any indirect revenue effects all discounted at the pre-tax rate of return and mulitiplied by the MCF. This is the MCF criterion proposed by Liu (2003).

Liu argues that both the SOC criterion and the STP criterion are flawed. He maintains that the SOC criterion proposes to discount both benefits and costs at the social opportunity cost of capital, here identified by ρ, while ignoring any indirect revenue effects. If this were true it would clearly conflict with the MCF criterion, even in situations where indirect revenue effects were assumed to be zero. On the other hand, the STP criterion proposes to convert all investment displaced or induced by the project into its consumption equivalent and discount at the after tax rate of return. For projects with deferred costs as well as deferred benefits the STP criterion would conflict with the MCF criterion even when there are no indirect revenue effects and the MCF happened to equal the appropriate conversion factor.

I aim to show that Liu's claims are incorrect. Both the SOC criterion and the STP criterion are equivalent to the MCF criterion if they are implemented properly. To facilitate the argument I will focus on two special cases that have drawn the most attention in the literature: project benefits being fully consumed, and project benefits being treated as equivalent to income.

2.1 Benefits Fully Consumed

A project may or may not affect private sector behaviour, even though the private sector has a willingness to pay for the project's output. Given the additively separable intertemporal structure of the utility function, an increase in g^t has no effect on the marginal utility of private consumption in time periods other than period t. If we impose the additional restriction that $\partial^2 U/\partial c^t \partial g^t = 0$, the project has no effect on the marginal utility of private consumption in period t. Then an increase in g^t will leave the time path of private consumption unaffected, i.e. $\partial c^i/\partial g^t = 0$, for all i. The project's benefits will be "fully consumed" in the period in which they are produced so the project leaves no trace in terms of private consumption, a property known as "uncompensated independence".[4] A project that has no impact on private consumption will have no impact on saving, and therefore no impact on assets or capital income tax revenue. In other words, the indirect revenue effects of the project will be zero. Setting $IR^t = 0$ in equation (4), the MCF criterion simplifies to

$\Sigma_{t=1}^{\infty} B^t/(1+r)^t - MCF.dI^0 > 0$

For a project requiring an initial expenditure of dI^0 that produces a perpetual stream of benefits worth B beginning in period 1, with the benefits being fully consumed, the MCF criterion simplifies to

$(5) - dI^0.MCF + B/r > 0.$

How should the SOC criterion be applied in this situation? The SOC criterion proposed by Harberger (1973) and Sandmo-Dreze (1971) requires that all benefits and costs (including any "indirect revenue effects") be discounted at a rate reflecting the social opportunity cost of capital.[5] The benchmark for

[4] Wildasin (1984) emphasizes the distinction between compensated and uncompensated independence in deriving criteria for the optimal supply of a public good in a static context. See also Browning (1987).

[5] Harberger (1973, pp. 47-48) explicitly states that whenever the project induces a shift in demand or supply in any market that is distorted by a tax, the magnitude of the shift should be multiplied by the tax wedge and the resulting indirect revenue effects should be added to (or subtracted from) net benefits, which are discounted at the SOC rate. Sjaastad and Wisecarver (1977, p. 533) also emphasize that Harberger's SOC rate refers only to the raising of funds, while acknowledging that how the funds are spent can affect private sector decisions. Sandmo-Dreze (1971) do not explicitly address indirect revenue effects, but their specification

measuring indirect revenue effects using the SOC criterion is a project whose benefits are a perfect substitute for income, i.e. a project whose impact on private sector behaviour is equivalent to the impact of an increase in income equal to the project's benefits. This is a project for which $\partial c^i/\partial g^t = p_g^t \partial c^i/\partial y^t$ for all i, where y^t is income in period t and p_g^t is the marginal willingness to pay for a unit of the project's output in period t. But according to the Slutsky equation, $\partial c^i/\partial g^t = (\partial c^i/\partial g^t)_u + p_g^t \partial c^i/\partial y^t$, so the SOC criterion takes as its benchmark a project for which the compensated effect on consumption in all periods is zero, i.e. $(\partial c^i/\partial g^t)_u = 0$, for all i.

In applying the SOC criterion, indirect revenue effects are defined as the **compensated** effect of the project on capital income tax revenue. Thus the indirect revenue effect in period i of a project that yields output of dg^t in periods $t = 1, 2, ...\infty$ is the impact of the project on capital income tax revenue in period i. $IR_{soc}^i = -\tau\rho\Sigma_{t=1}^{\infty}(\partial c^i/\partial g^t)_u dg^t$

If the project's benefits are in fact fully consumed, then $\partial c^i/\partial g^t = 0$ for all i. Therefore $(\partial c^i/\partial g^t)_u = -p_g^t \partial c^i/\partial y^t = -p_g^t(\partial c^i/\partial y_0)/(1+r)^t < 0$, so the compensated effect of the project on consumption is negative in all periods; the project will reduce consumption in all periods relative to the benchmark. Since any reduction in consumption is an increase in saving (and therefore assets), the indirect revenue effects of the project will be positive. Specifically, the indirect revenue effect of the project in period i is

$IR_{soc}^i = -\tau\rho\Sigma_{t=1}^{\infty}(\partial c^i/\partial g^t)_u dg^t = \tau\rho\Sigma_{t=1}^{\infty}\left[p_g^t(\partial c^i/\partial y_0)/(1+r)^t\right] dg^t = \tau\rho B/(1+r)$.[6]

Intuitively, if the project produces benefits worth B in perpetuity starting in period 1, and if these benefits are fully consumed, the project will reduce private consumption (increase saving) by $B/(1+r)$ relative to the benchmark, i.e. relative to a project whose benefits are treated as equivalent to income. Therefore, relative to the SOC benchmark the project will generate additional capital income tax revenue equal to $\tau\rho B/(1+r)$ in all subsequent periods.[7] The SOC criterion applied to this project then becomes

(6) $-dI^0 + B/\rho + IR_{soc}/\rho > 0$

Benefits and costs, including indirect revenue effects, should be discounted at the SOC rate.

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assumes that the compensated impact of the project on private consumption (and therefore saving) is zero. See also Burgess (1988).

[6] Since the project yields a constant stream of output, $dg^t = dg$ for all t, and $p_g^t dg^t = B^t = B$ for all t by assumption. Finally, $\partial c^i/\partial y_0 = r/(1+r)$ for all i assuming that the representative agent consumes the annuity value of wealth. Making these substitutions results in the last equality.

[7] A project with benefits worth B starting in period 1 and continuing in perpetuity will increase wealth by B/r if these benefits are regarded as a perfect substitute for income. Assuming the representative agent consumes the annuity value of wealth, her consumption will increase in each period starting in period 0 by $B/(1+r)$ and capital income tax revenue will decrease in each period starting in period 1 by $\tau\rho B/(1+r)$. Since the project's benefits are fully consumed, it will **increase** capital income tax revenue in each period starting in period 1 by $\tau\rho B/(1+r)$ compared to the benchmark that is used for the SOC criterion, i.e. a project whose benefits are a perfect substitute for income.

To confirm that the SOC criterion is equivalent to the MCF criterion, substitute for $IR_{soc} = \tau\rho B/(1+r)$ to obtain the condition

$-dI^0 + B/\rho + \tau\rho B/(1+r)\rho > 0$

which simplifies to

$-dI^0 + B(1+\rho)/\rho(1+r) > 0$

This can be re-written as

$-dI^0.MCF + B/r > 0$

which is the MCF criterion arrived at in equation (4). Benefits should be discounted at the after-tax rate, whereas costs should be discounted at the pre-tax rate but multiplied by the MCF.

2.2 Benefits Treated as Income

The assumption that the project has no impact on private sector behaviour is very special. Indeed, it can be argued that if a person who experiences a change in the level of a publicly provided good takes no actions whatsoever in response the publicly provided good is not a significant source of value for the person.[8] More plausible is the notion that the publicly provided good $g^t$and contemporaneous private consumption c^t are substitutes rather than independents or complements. An increase in g^t then lowers the marginal utility of private consumption in period t leaving the marginal utility of private consumption in all other periods unchanged. In an effort to smooth consumption, the individual responds by reducing private consumption in period t and increasing private consumption in all other periods. If g^t and c^t were complements, the individual would respond by increasing consumption in period t and reducing consumption in all other periods which seems counter-intuitive for an individual striving to smooth consumption over her lifetime.

Suppose the utility function takes the form $\Sigma_{t=0}^{\infty}\beta^t U(c^t, q(e^t, g^t))$ where $U(.)$ and $q(.)$ are monotonic increasing and concave functions of their arguments. In this formulation private expenditure in period t consists of two components: ordinary private consumption c^t, and averting expenditure e^t that is motivated solely by the desire to mitigate the adverse effects of the limited supply of the publicly provided good g^t. One can imagine the publicly provided good g^t serving as an input along with a private good e^t in the "household production" of q^t. The formulation therefore allows g^t and e^t to be imperfect substitutes in the production of q^t.[9] An increase g^t will increase c^i in all periods including period t, increase e^i in all periods except period t, but reduce e^t and "private spending" in period t (defined as $c^t + e^t$). If sufficient income is then taken away to keep utility fixed, c^i and e^i will be unchanged for all i and only e^t will decrease, with the decrease in e^t being equal to the reduction in income required to keep utility fixed.[10] In other words, $(\partial c^i/\partial g^t)_u = \partial c^i/\partial g^t - p_g^t(\partial c^i/\partial y_0)/(1+r)^t = 0$, for all

[8] See e.g. Larson (1993).

[9] The special case of g^t being a perfect substitute for some private good e^t is included in this specification, but it is unnecessarily restrictive. See Liu et al (2005).

[10] This is an extension to an inter-temporal context of the model of defensive spending formulated by Courant and Porter (1981) and Bartik (1988).

i. The private sector will respond to an increase in the publicly provided good g^t in the same way as it would to an increase in a lump sum transfer equal to the public's willingness to pay for g^t. Since the compensated response in c^i to an increase in g^t is zero for all i, the uncompensated response must be positive. Specifically, $\partial c^i/\partial g^t = p_g^t(\partial c^i/\partial y_0)/(1+r)^t > 0$ for all i.

In applying the MCF criterion, the indirect revenue effect is the **uncompensated** effect of the project on capital income tax revenue. Thus $IR^i = -\tau\rho\Sigma_{t=1}^{\infty}(\partial c^i/\partial g^t)dg^t = -\tau\rho\Sigma_{t=1}^{\infty}\left[p_g^t(\partial c^i/\partial y_0)/(1+r)^t\right]dg^t = -\tau\rho B/(1+r)$ when $dg^t = dg$, for all t.[11]

Intuitively, the project's benefits are equivalent to income worth B each period beginning in period 1, so the project will increase consumption in period 0 and thereafter by $dc^t = B/(1+r)$, and decrease assets in period 1 and thereafter by $dA^t = -B/(1+r)$. Since the income on assets is subject to tax at rate τ the indirect revenue effect of the project in periods $t = 1, ...\infty$ is $IR^t = -\tau\rho B/(1+r)$. The MCF criterion applied to a project whose benefits are equivalent to income becomes

(7) $-\left\{dI^0 + \tau B/(1+r)\right\}MCF + B/r > 0$

To confirm that the MCF criterion represented by (7) correctly identifies all welfare improving projects, recall that a project is worthwhile if (1) holds, and the project is fiscally feasible if it satisfies the government budget constraint (2). But $dR^t = \tau\rho dA^t$, and $dA^t = -(B + dT^0)/(1+r)$. Therefore, to satisfy (2) the lump sum tax increase must be $dT^0 = \left\{dI^0 + \tau B/(1+r)\right\}\rho(1+r)/r(1+\rho)$. Substitute this expression into (1) and recall that the MCF for a lump sum tax is $MCF = \rho(1+r)/r(1+\rho)$. The result is (7).

The MCF criterion expressed in equation (7) can be further simplified by substituting for $MCF = \rho(1+r)/r(1+\rho)$. Making this substitution and rearranging terms we see that the project is worthwhile provided that

(8) $-dI^0 + B/\rho > 0$

This is the SOC criterion proposed by Harberger (1973) and Sandmo-Dreze (1971). Simply put, the representative agent is better off with the project as long as its benefits discounted at the SOC rate exceeds its costs. There are no "indirect revenue effects" to take into account when the project's benefits are equivalent to income.

To summarize, there is no conflict between the SOC criterion and the MCF criterion. The criteria differ in terms of the benchmark that is used to measure indirect revenue effects. While the choice of benchmark is a matter of taste, the reality is that indirect revenue effects are difficult to measure (and therefore typically ignored in project evaluation). This being the case, it is best to choose a criterion where ignoring indirect revenue effects is least objectionable. On these grounds, there are at least four situations where the SOC criterion would seem to be the preferred choice.

[11] Since $IR^i_{soc} = IR^i - \tau\rho B/(1+r)$ if $IR^i = 0$ then $IR^i_{soc} = -\tau\rho B/(1+r)$ and if $IR^i_{soc} = 0$ then $IR^i = \tau\rho B/(1+r)$. Thus the indirect revenue effect that enters into the MCF criterion is precisely equal, but opposite in sign, to the indirect revenue effect that enters into the SOC criterion.

First, if a project yields benefits that are appropriable via user fees or normal market transactions, (i.e. benefits that the private sector can provide) the appropriate discount rate is the SOC rate. Projects like electricity generation or water and wastewater treatment services would fall into this category.

Second, for projects that yield benefits that the private sector appropriates as a component of full income the appropriate discount rate is the SOC rate. Education and training programs that improve labour skills thereby raising real wages, or infrastructure projects whose benefits are reflected in higher land values and rents would fall into this category.

Third, the SOC rate is appropriate for projects that provide services that are a perfect substitute for some privately produced good such as "free" school lunch programs, or public health care that reduces the demand for private health care.

Fourth, if individuals purchase some market goods to mitigate the adverse effects of environmental bads, the benefits of an increment in government spending to improve environmental quality can be measured by the reduction in private spending on mitigation. Examples include the purchasing water filters or air purifiers to defend against poor air or water quality.

In each of these situations an increase in the publicly provided good is equivalent to an equal value increase in lump sum transfers insofar as its impact on private consumption is concerned. Therefore for a project that is just worth doing, any impact of the project on capital income tax revenue will be offset by the impact of the lump sum tax increase needed to finance the project.

2.3 Shadow Pricing Algorithm

Now let us look at the social rate of time preference approach (STP) proposed by Marglin (1964), Feldstein (1972), Bradford (1975) and Lind (1982). The procedure is to convert all benefits and costs into "consumption equivalents" by shadow pricing all investment displaced or induced and discount at the social rate of time preference (which I will interpret as the after tax rate of return). For a project requiring an initial expenditure of dI^0 and producing a stream of output dg in each subsequent period worth $B(dg)$ with these benefits being "fully consumed", the STP criterion judges the project to be worthwhile provided that

(9) $-dI^0((1-\gamma)+\gamma SPC))+B(dg)/r>0.$

Here γ is interpreted as the proportion of project financing that displaces private investment (so $(1-\gamma)$ is the proportion that displaces consumption), and SPC is the "shadow price of capital", which is defined the present value of the stream of consumption that is lost when a dollar of private investment is displaced. Since private investment yields a pre-tax rate of return of ρ and the representative agent consumes the annuity value of wealth, a dollar's worth of private investment is worth ρ/r in terms of foregone consumption.

Clearly, the shadow price of capital criterion is equivalent to the MCF criterion given in equation (5) only if the MCF happens to equal $(1-\gamma)+\gamma SPC$. Most practitioners have assumed that γ represents the marginal propensity to save.[12] Thus, if the government must raise lump sum taxes to finance the project γ is the proportion of the lump sum tax increase that comes from reduced saving and $1-\gamma$ is the proportion that comes from reduced consumption.[13] If the representative agent consumes the annuity value of wealth, the proportion of a lump sum tax increase that comes from reduced consumption will be $r/(1+r)$. It is then inferred that the proportion of the **project's expenditure** that comes from consumption is $(1-\gamma) = r/(1+r)$ and the proportion that comes from investment is $\gamma = 1/(1+r)$.

But this is incorrect. If the cost of the project is dI^0 the government must raise lump sum taxes by $dT^0 = MCF.dI^0$ to finance the project, so the amount of resources drawn from consumption is $\gamma dT^0 = rdT^0/(1+r) = MCF.dI^o.r/(1+r) = \rho dI^0/(1+\rho)$, and therefore the amount drawn from investment is $dI^0/(1+\rho)$.[14] Using these proportions it is easy to confirm that $(1-\gamma)+\gamma SPC = MCF$. Thus, the present value of the consumption foregone per dollar of resources diverted to the project is equal to the marginal cost of funds, i.e. the ratio of the increase in lump sum taxes to the increase in the present value of total tax collected.

Liu (2003) claims that the STP criterion is flawed, but he provides no intuition as to why. I have shown that the problem arises because of a failure to distinguish between the expenditure requirements of the project and cost of financing the expenditure requirements. Given the pre-existing capital income tax distortion, the present value of the increase in lump sum taxes necessary to finance the project will exceed the project's expenditure requirements. In order to determine the proportion of resources drawn from consumption one must multiply the marginal propensity to consume by the MCF. If γ is interpreted as the proportion of the project's expenditure requirement that displaces investment (rather than the proportion of the cost of financing the project that displaces investment) then the shadow pricing algorithm turns out to be perfectly consistent with the MCF criterion: both correctly identify projects that make the infinitely lived representative agent better off.[15]

If the project's benefits were treated as income instead of being "fully con-

[12] See e.g. Bradford (1975), Mendelshon (1981) and Lind (1982).

[13] Note that the government must increase lump sum taxes to finance the project since the project's benefits are not treated as income, so any debt issued to finance the project would be a future tax liability of equal present value.

[14] The government could raise lump sum taxes by $dT^0 = dI^0$ in period 0 to maintain a balanced budget, but it would subsequently have to raise lump sum taxes further to compensate for the loss in capital income tax revenue. The **present value** of the lump sum tax increase necessary to finance the project is therefore $dT^0 = MCF.dI^0$. The rational agent with perfect foresight will anticipate the tax increase required to finance the project and adjust her consumption and saving accordingly.

[15] The 'equivalence' result is robust to more complicated projects. Thus for a project that requires expenditures of dI^0 and $dI^1$and produces benefits worth B beginning in period 2, the procedure is to first calculate the present value of the project's expenditure requirements (discounted at the pre-tax rate), namely $dI^0+dI^1/(1+\rho)$ then convert this into its 'consumption equivalent' by multiplying by $1-\gamma+\gamma SPC$ where $SPC = \rho/r$ and $\gamma = 1/(1+\rho)$. If the project's benefits are 'fully consumed' the project is worthwhile according to the shadow pricing algorithm if $B/r(1+r) - (dI^0 + dI^1/(1+\rho))(1-\gamma+\gamma SPC) > 0$. This is equivalent to the MCF criterion.

sumed", the STP criterion in equation (9) would simplfy to

$-dI^0 SPC + B(dg)/r > 0$

Since the benefits are treated as income, financing the project would displace private investment dollar for dollar whenever the pre-tax rate of return ρ is exogenous. Since $SPC = \rho/r$ the STP criterion is equivalent to the SOC criterion, a point made by Sjaastad and Wisecarver (1977). On the other hand, the MCF criterion in this situation is given in equation (7), which amounts to the same thing.

To summarize, using an infinitely lived representative agent model I have reconciled 3 criteria for project evaluation that are prominent in the literature. It turns out that if the criteria are appropriately applied they are consistent with each other; they all correctly identify projects that make the infinitely lived representative agent better off.

3 Overlapping Generations Model

The ILA model cannot address issues of inter-generational equity, which frequently arise in discussions about the appropriate social discount rate. Specifically, individuals alive when the project's costs are incurred are not necessarily alive to enjoy the benefits. To explicitly recognize the inter-generational implications of public investment decisions I will assume a simple overlapping generations (OLG) model in which individuals live for two periods, working, consuming and saving in their youth, and living off their assets in old age. There are no bequests. In order to judge whether a project is worthwhile we need a welfare criterion. Because I want to separate project assessment from issues of inter-generational redistribution, I assume that a project is worthwhile only if a Pareto improvement is possible. In other words, all generations must be at least as well off with the project as without it, and at least one generation must be better off. For the marginal project, all generations must be just as well off. I show in the context of the OLG model that the SOC criterion and the MCF criterion both satisfy this requirement, but the social rate of time preference criterion (STP) does not.

Suppose an individual born in period t has the utility function

$U(c_1^t, g^t) + \beta U(c_2^t)$

where β represents the individual's pure rate of time preference. In this formulation the publicly provided good is assumed to be beneficial only to the young.

As in the ILA model, the pre-tax wage w and rate of return ρ are exogenous and there is a tax on capital income at rate τ, so $\rho(1-\tau) = r$. The individual's budget constraint states that the present value of consumption equals wage income minus lump sum taxes, where the discount rate equals the after tax rate of return.Thus

$c_1^t + c_2^t/(1+r) = w - T^t$

The first order conditions determine the ordinary demand functions

$c_1^t(g^t, T^t),\ c_2^t(g^t, T^t) = (1{+}r)s^t(g^t, T^t) = (1{+}r)(w{-}T^t{-}c_1^t)$, so an individual born in period t chooses her intertemporal consumption plan conditional on the available supply of the publicly provided good g^t and the lump sum tax T^t.

Now consider a project that requires an initial expenditure of dI^0 and yields outputs in the subsequent two periods of $dg^1$and dg^2 worth $B^1 = B(dg^1)$ and $B^2 = B(dg^2)$. The benefits reflect what the subsequent two young generations are willing to pay for the project's output in terms of contemporaneous consumption.

If the project is to leave those currently alive no worse off it must be debt financed. But a debt financed project will displace private investment dollar for dollar because the pre-tax rate of return is exogenous. If the project is to leave those living in the next two periods no worse off, the benefits they enjoy must be at least as great as the lump sum tax increase they incur. Subsequent generations receive no benefits, so they will be indifferent to the project as long as their lump sum tax is unaffected. Meanwhile, the project is feasible if the present value of the increase in project expenditures is equal to the present value of the increase in taxes, where the discount rate is the pre-tax rate of return.

Let $V(g^t, T^t)$ be the well-being of the young generation living in period $t = 1, 2$. The project will leave them just as well off if $(\partial V/\partial g^t)dg^t + (\partial V/\partial T^t)dT^t = 0$. Since $\partial V/\partial g^t = (\partial U/\partial g^t)$ and $\partial V/\partial T^t = -\partial U/\partial c_1^t$, generation t will be at least as well off with the project if

$(\partial U/\partial g^t)/(\partial U/\partial c_1^t)dg^t - dT^t \geq 0$

The coefficient on dg^t is the marginal rate of substitution between g^t and c_1^t, i.e. the individual's marginal willingness to pay for a small increment in the publicly provided good. Therefore, the project will be worthwhile provided that

$(*)B^t - dT^t \geq 0, t = 1, 2.$

Taxes in period t reflect lump sum taxes paid by the young plus capital income taxes on the assets (saving) of the old so $R^t = T^t + \tau\rho s^{t-1}$. Note that since the project is debt financed it will have no impact on saving in period 0, and since benefits accrue only to the young in periods 1 and 2 there must be no change in lump sum taxes in periods 3 and beyond. Therefore the project will satisfy the government's budget constraint provided that

$(**) - dI^0 + dT^1/(1+\rho) + (dT^2 + \tau\rho ds^1)/(1+\rho)^2 + (\tau\rho ds^2)/(1+\rho)^3 = 0.$

But $s^t(g^t, T^t)$, so $ds^t = (\partial s^t/\partial g^t)dg^t + (\partial s^t/\partial T^t)dT^t$, $t = 1, 2$. Therefore, substituting $(*)$ into $(**)$ the project will be worthwhile provided that

$(10){-}dI^0{+}B^1/(1{+}\rho){+}B^2/(1{+}\rho)^2{+}\tau\rho\left\{(\partial s^1/\partial g^1)dg^1 + (\partial s^1/\partial T^1)dT^1\right\}/(1{+}$ $\rho)^2 + \tau\rho\left\{(\partial s^2/\partial g^2)dg^2 + (\partial s^2/\partial T^2)dT^2\right\}/(1+\rho)^3 \geq 0$

Note that $-\ \partial s^t/\partial T^t$ represents generation $t's$ marginal propensity to save. For simplicity assume the marginal propensity to save is the same for all generations so $-\partial s^t/\partial T^t = \gamma$. Since a worthwhile project must satisfy $B^t - dT^t \geq 0, t = 1, 2$, and focusing on the marginal project that is just worthwhile we can substitute $(\partial s^t/\partial T^t)dT^t = -\gamma B^t, t = 1, 2$.

The indirect revenue effect of the project according to the MCF criterion is the impact of the project on capital income tax revenue. Specifically $IR^t = \tau\rho(\partial s^t/\partial g^t)dg^t/(1+\rho)$. If the output of the project in period t affects saving in period t it will affect the capital income tax revenue collected in period $t+1$.The indirect revenue effect of the project in period t is the impact on capital income tax revenue collected evaluated in period t, which explains why the term is divided by $1+\rho$.

Making all the above substitutions in equation (10) we arrive at the condition

(11)$-dI^0+IR^1/(1+\rho)+IR^2/(1+\rho)^2+\left[B^1/(1+\rho)+B^2/(1+\rho)^2\right][1-\tau\rho\gamma/(1+\rho)] \geq 0$

The last term in square brackets represents the increase in the present value of tax revenue collected per dollar increase in lump sum tax. The reciprocal represents the marginal cost of funds for a lump sum tax. Thus, a project is just worthwhile (i.e. enables all generations to be just as well off) if the present value of its expenditure requirements minus its indirect revenue effects discounted at the pre-tax rate of return and multiplied by the MCF equals the present value of its benefits discounted at the pre-tax rate of return. This is the MCF criterion proposed by Liu et al (2004).

These authors argue that the SOC criterion, which proposes to discount benefits and costs at the SOC rate, is flawed for two reasons. First, it ignores the fact that the MCF is greater than one for a lump sum tax. Second, it looks only at the project's direct costs and benefits and ignores the existence of indirect revenue effects. Reasoning from a version of the MCF criterion similar to equation (11), they maintain that the appropriate social discount rate will not be equal to the social opportunity cost of public funds (here represented by ρ) except under very special circumstances. For a project with no indirect revenue effects they claim that the appropriate social discount rate will exceed ρ because MCF exceeds unity. In general, the appropriate social discount rate will be "project specific".

All of these claims are false. They reflect a misunderstanding about how to implement the SOC criterion. The appropriate social discount rate is indeed the social opportunity cost of public funds, here represented by ρ, and it is project independent. The SOC criterion recognizes the possibility of indirect revenue effects, but compared to the MCF criterion indirect revenue effects reflect the **compensated** impact of the project on capital income taxes rather than the **uncompensated** impact. Finally, the marginal cost of funds exceeds unity for a lump sum tax but it is **irrelevant** in implementing the SOC criterion.

Recall that the uncompensated impact of the project with output dg^t on capital income tax revenue valued in period t is $IR^t = \tau\rho(\partial s^t/\partial g^t)dg^t/(1+\rho)$. Therefore the compensated impact of the project is $IR^t_{soc} = \tau\rho\left[\partial s^t/\partial g^t - p_g \partial s^t/\partial y^t\right] dg^t/(1+\rho)$. Since $\partial s^t/\partial y^t = \gamma$, and $p_g dg^t = B(dg^t)$, we can write

$IR^t_{soc} = IR^t - \tau\rho\gamma B^t/(1+\rho)$.

Now use this expression to replace IR^t in equation (11) to obtain

(12)$-dI^0+IR^1_{soc}/(1+\rho)+IR^2_{soc}/(1+\rho)^2+\left[B^1/(1+\rho)+B^2/(1+\rho)^2\right][1-\tau\rho\gamma/(1+\rho)] \geq 0$

This simplifies to

(13) $-dI^0+IR^1_{soc}/(1+\rho)+IR^2_{soc}/(1+\rho)^2+B^1/(1+\rho)+B^2/(1+\rho)^2 \geq 0$

which is the SOC criterion. Benefits and costs, including indirect revenue effects, are discounted at a rate that reflects the social opportunity cost of capital.

3.1 Two Special Cases

Now let us return to the two special cases previously discussed: project benefits treated as income; and project benefits fully consumed.

Suppose the individual's utility function takes the form

$U(c_1^t, q(g^t, e_1^t)) + \beta U(c_2^t)$

Here g^t is not consumed directly, but rather serves as an input along with a component of private spending e_1^tin the "household production" of q^t.

Then an increase in project output of dg^t accompanied by an increase in lump sum taxes dT^t equal to the individual's willingness to pay for dg^t will leave $c_1^t$and c_2^t unchanged and reduce e_1^t by $de_1^t = -dT^t$. The project's benefits are regarded as a perfect substitute for income. The compensated effect of the project on saving and therefore capital income tax revenue will be zero, so the indirect revenue effects relevant for the SOC criterion will be zero. Thus, setting the terms in braces in equation (10) equal to zero, or equivalently setting $IR_{soc}^t = 0$ for $t = 1, 2$ in equation (13) we have the condition

(14) $-dI^0 + B^1/(1+\rho) + B^2/(1+\rho)^2 \geq 0$

which is the SOC criterion.

But recall that $IR_{soc}^t = IR^t - \tau\rho\gamma B^t/(1+\rho)$. Therefore $IR^t = \tau\rho\gamma B^t/(1+\rho) > 0$. Substitute for IR^t in equation (11) and we have a version of the MCF criterion that reduces to (14).

Next, suppose that the individual's utility function, in addition to being additively separable intertemporally, has the property that $\partial^2 U/\partial c_1^t/\partial g^t = 0$. Then an increase in g^t has no impact on the ordinary (uncompensated) demands for private goods, and therefore no impact on saving or capital income tax revenue. The indirect revenue effects that are relevant for the MCF criterion are therefore equal to zero. Setting $IR^t = 0$ for $t = 1, 2$ in equation (11) we have the simplified MCF criterion

(15) $-dI^0 + \left[B^1/(1+\rho) + B^2/(1+\rho)^2\right][1 - \tau\rho\gamma/(1+\rho)] \geq 0$

Thus the project's benefits and costs should be discounted at the pre-tax rate of return, but the costs should be multiplied by the MCF.

But since $IR_{soc}^t = IR^t - \tau\rho\gamma B^t/(1+\rho)$, whenever $IR^t = 0$ then $IR_{soc}^t = -\tau\rho\gamma B^t/(1+\rho) < 0$. Substitute for IR_{soc}^t in equation (13) and we have a version of the SOC criterion that reduces to (15).

Thus once again we see that there is no conflict between the SOC criterion when it is implemented correctly and the MCF criterion proposed by Liu et al (2004). As noted by Ballard and Fullerton (1992) there is ambiguity in the literature regarding the meaning of the marginal cost of funds. According to the Pigou-Harberger-Browning tradition the benchmark is a project characterized by compensated independence and the MCF is equal to unity for a lump sum tax because the tax does not shift the compensated demand (supply) for the tax distorted good (factor). According to the Stiglitz-Dasgupta-Atkinson-Stern tradition, the benchmark is a project characterized by uncompensated independence and the MCF can be greater or less than one for a lump sum tax depending on the tax regime. The tax shifts the uncompensated demand (supply) for the tax distorted good (factor) but the spending of tax revenue has an offsetting income effect. Liu et al (2004) follow the latter approach in arriving at their MCF criterion. I have followed the former approach and arrived at the SOC criterion, which amounts to the same thing.

3.2 Shadow Pricing Algorithm

Finally, let us consider the shadow pricing algorithm proposed by Marglin, Feldstein, Bradford and Lind. All benefits and costs must be converted into "consumption equivalents" by shadow pricing any investment displaced or induced by the project and then discounted at the social rate of time preference (after tax rate of return). If the benefits are fully consumed the criterion for a worthwhile project is

$-dI^0((1-\gamma)+\gamma SPC)+B^1/(1+r)+B^2/(1+r)^2>0$

where γ is the proportion of funding that displaces private investment and SPC is the shadow price of capital. In the OLG model a dollar's worth of investment produces $1+\rho$ dollar's worth of consumption in the next period so $SPC=(1+\rho)/(1+r)$. Also, since the project yields benefits only to young generations living in the subsequent two periods the project must be debt financed to avoid an inter-generational transfer from current to future generations. Furthermore, since the pre-tax rate of return is exogenous the funding will displace private investment dollar for dollar so $\gamma=1$. In the present example the STP criterion becomes

$-dI^0(1+\rho)/(1+r)+B^1/(1+r)+B^2/(1+r)^2>0$

This differs from the MCF criterion for a project whose benefits are fully consumed given in equation (15). The shadow pricing algorithm discounts benefits at r rather than ρ, which therefore overstates the benefits, but it also multiplies costs by a shadow price of capital that exceeds the marginal cost of funds thereby overstating costs. Thus it is not possible to rank the STP criterion versus the MCF criterion in this example. All that can be said is that the internal rates of return on the marginal project differ according to the two criteria. However, since we have already shown that the MCF criterion is necessary and sufficient for a Pareto improvement to be possible we can conclude that the STP criterion fails to meet this standard.

What if the benefits are treated as income rather than being "fully consumed"? Specifically, what if each generation that receives benefits alters its intertemporal consumption plan by consuming only part of the benefits while young and saving the remainder for old age? The STP criterion then becomes

$-dI_0(1+\rho)/(1+r)+B^1\left[(1-\gamma)+\gamma SPC\right]/(1+r)+B^2\left[(1-\gamma)+\gamma SPC\right]/(1+r)^2>0$

Once again, the project must be debt financed to avoid burdening those currently alive, so the funding will displace private investment dollar for dollar. But those living in the subsequent two periods will treat the project's benefits as income and save a proportion γ, with each dollar saved having a "consumption equivalent" value of $(1+\rho)/(1+r)$.

It is easy to confirm that the STP criterion will judge a project as worthwhile even though it fails to satisfy the SOC criterion, and therefore the STP criterion fails to ensure that a Pareto improvement is possible.

To emphasize this point, consider an investment costing dI^0 and yielding benefits worth B in all subsequent periods. Assume that these benefits are treated as income. If the project is debt financed the STP criterion would judge the project as worthwhile if

$-dI^0(1+\rho)/(1+r) + B\left[1-\gamma+\gamma(1+\rho)/(1+r)\right]/r > 0$

whereas the SOC criterion would judge the project as worthwhile if

$-dI^0 + B/\rho > 0.$

Suppose $\rho = .1, r = .03$, and $\gamma = .25$.Then the STP criterion would judge the project as worthwhile provided that its internal rate of return was greater than 3.15% even though the resources to finance the project are drawn from private investment that would have earned 10%.

3.3 The SOC Criterion without the Compensation Test

Those who advocate the STP criterion are explicitly or implicitly dismissing the relevance of the Kaldor-Hicks compensation test. They are assuming that a social welfare gain can occur even if a Pareto improvement is not possible. Specifically, a project can be worthwhile even if those currently alive are made worse off, provided that for every dollar of consumption foregone by the current generation the next generation's consumption increases by at least $1+r$ dollars, or the subsequent generation's consumption increases by $(1+r)^2$, etc. However, satisfying the STP criterion is still not sufficient to recommend the project because other more worthy investment opportunities may be available.

To emphasize this point, consider a simple one period project that costs dI^0 and produces benefits worth B^1 to the next young generation who will treat the benefits as income. Suppose the project is tax financed, and let γ represent the proportion of tax revenue drawn from saving. The STP criterion would judge the project as worthwhile provided that

$B^1\left[1-\gamma+\gamma(1+\rho)/(1+r)\right]/(1+r) > dI^0\left[1-\gamma+\gamma(1+\rho)/(1+r)\right].$

The expression on the left hand side represents the present value of the stream of consumption induced by the project, and the expression on the right hand side represents the present value of the stream of consumption displaced when the project is financed. The STP criterion judges the project as worthwhile because it induces a more valuable consumption stream than it displaces. The project yields a "social welfare improvement" according to the social welfare function that has been (implicitly) adopted. Since the expression in squared brackets is common to both sides, the STP criterion judges the project to be worthwhile provided that $B^1/(1+r) > dI^0$. The project must have an internal rate of return in excess of the social rate of time preference (after tax rate of return) to be worthwhile.

By contrast, the SOC criterion would require that the project's internal rate of return exceed the social opportunity cost of capital in order to be worthwhile.

$B^1/(1+\rho) > dI^0$

Note that this is true whether the project is financed by increasing lump sum taxes or by borrowing.

Before we undertake the project on the strength of the STP criterion, we should check to see whether there are alternative uses of the funds that could make everyone even better off.

Proponents of the STP criterion recognize that there may be higher return investments available in the private sector, but they rule these out on the grounds that private sector investments are not feasible alternatives for the government to pursue on political grounds. "Government investment is restricted to certain classes of activity", according to Bradford (1975, p. 888), and "significant areas of economic activity are reserved for private enterprise", according to Lind (1982, pp. 30-31). These authors seem to think that with the tax structure taken as given, the only way the government can encourage private investment using the tax dollars available is to invest directly in the private sector, or to subsidize particular private sector activities. But there is a third option: use the tax dollars to pay down the debt.

The government's budget constraint in period t can be written as:

$D^1 = (1+r)D^0 + G - \tau\rho K^0 - T^0.$

Here D^0 represents government debt at the beginning of period 0, G represents statutory social security benefits paid to the retired generation (which I shall assume remains fixed throughout). Clearly, if the government increases lump sum taxes in period 0 and uses the funds for debt reduction it will reduce the government debt outstanding at the beginning of period 1 by an equal amount (because all other variables in the equation are pre-determined). Thus, $\Delta D^1 = -\Delta T^0$. Now consider the assets-saving relationship written as: $A^1 = s^0 = \gamma(w - T^0)$. Here A^1 represents the economy's assets at the beginning of period 1. Assets in any period consist of private capital and government debt so $A^1 = K^1 + D^1$. According to the assets-saving relationship, an increase in the lump sum tax in period 0 that is used to reduce the government's borrowing in period 1 will increase private investment in period 1 by $\Delta K^1 = (1-\gamma)\Delta T^0$. The change in private investment is the sum of two components: the reduction $-\gamma\Delta T^0$ that occurs because the tax reduces the pool of saving, and the increase ΔT^0 that occurs because the tax increase is used to reduce government debt. Thus, if the additional tax dollars raised in period 0 are used to pay down the debt rather than to finance the project, private investment will increase in period 1 by exactly the amount that government borrowing is reduced.

To compare the project to the alternative of using the tax dollars for debt reduction I assume that the government returns all the benefits of debt reduction to the next working generation by a tax cut. Specifically, it reduces the lump sum tax in period 1 by the amount necessary for government borrowing in period 2 to equal its level in period 0. Since the government budget constraint in period 1 is: $D^2 = (1+r)D^1 + G - \tau\rho K^1 - T^1$ and since $\Delta D^2 = 0$ by assumption, then $\Delta T^1 = (1+r)\Delta D^1 - \tau\rho\Delta \bar{K}^1$. But when the additional tax dollars collected in period 0 are used for debt reduction $\Delta K^1 = -\Delta D^1$ so $\Delta T^1 = (1+\rho)\Delta D^1 = -(1+\rho)dI^0$. Thus the "debt reduction" project in period 0 that costs dI^0 dollars of tax revenue will allow the working generation in period 1 to enjoy a tax cut of $(1+\rho)dI^0$. Assuming that debt reduction is a feasible use of tax dollars, the benefits of any proposed project should be compared to the benefits of debt reduction. A project will benefit the next generation by more than debt reduction would only if $B^1 > dI^0(1+\rho)$. This is equivalent to the SOC criterion. In other words, **even if the project is tax financed, it should be evaluated as if the marginal source of funding were the capital market unless it can be shown that debt reduction is not a viable use of tax dollars**.

4 Concluding Remarks

Liu (2003) and Liu et al (2004) claim that the SOC criterion suffers from "severe implementation problems". Specifically, there is no general formula for the proportions of funding that are drawn from consumption and investment; the proportions depend upon the project, making the discount rate "project specific". These authors are interpreting the SOC as the internal rate of return on a project that is just worthwhile. By this methodology, when comparing two projects with identical costs and benefits, a project with positive indirect revenue effects, (i.e. a project that generates additional capital income tax revenue) will have a lower SOC than a project with no indirect revenue effects. However, if we follow Harberger (1973) and define the SOC as the "social opportunity cost of public funds", i.e. the rate of return foregone when the government borrows to finance a project whose benefits are equivalent to a lump sum transfer, the SOC rate will be unique and common to all projects. If there are indirect revenue effects they will be added to or subtracted from the benefits, and not incorporated by adjusting the SOC. Unlike the SOC criterion, Liu's MCF criterion requires no weights; all that is needed are estimates of the pre-tax and post-tax rates of return and the MCF, all of which are project independent. However, while the MCF is **project** independent, it is **model** dependent. In particular, the MCF will depend upon the pre and post-tax rates of return and the marginal propensity to save (among other things), and the marginal propensity to save will depend upon the time horizon and bequest motive of the representative agent. On the other hand, the weights needed to apply the SOC criterion will in general depend upon the elasticities of supply and demand for capital. It is therefore misleading to claim that the MCF criterion is exempt from implementation problems any more than is the SOC criterion.

The real difference between the two criteria is the type of project being evaluated. Are project benefits perceived as income or are they fully consumed, or are they neither? If benefits are treated as income then the SOC criterion has an implementation advantage because no indirect revenue effects need to be taken into account. If benefits are fully consumed then Liu's MCF criterion has an implementation advantage for the same reason. If benefits are neither fully consumed nor treated as income there will be indirect revenue effects to take into account using either criterion, and since there is a well defined relationship between the indirect revenue effects that apply to each criterion they will be equally difficult to measure in practical applications.

To summarize, there are 3 points to take away from this discussion.

First, the STP criterion proposed by Marglin, Feldstein, Bradford and Lind can be 'rescued' in the ILA model, but it is fundamentally flawed in the OLG model because it violates the compensation test and thereby fails to ensure that a Pareto improvement is possible. The STP criterion discounts benefits at the social rate of time preference (here equated to the after tax rate of return). However, whenever the benefits in different time periods accrue to different generations it is inappropriate to discount them at the after tax rate of return in judging whether a project can achieve a Pareto improvement. Even if the compensation test is deemed to be irrelevant, the STP criterion is still inappropriate because it leads to situations where a project may be judged worthwhile when there are alternatives that would make everyone even better off.

Second, Liu's criterion incorporating the MCF is sensitive to the underlying model. Not only is the MCF for a particular tax (e.g. a lump sum tax) model sensitive, but so is the appropriate social discount rate. Specifically, benefits are discounted at the pre-tax rate of return in the OLG model but at the after-tax rate of return in the ILA model. Thus in an economy with a given distortive tax on capital income a project with a given time stream of costs and benefits will be evaluated differently depending upon whether individuals have finite lives (and make no bequests) or act as if they live forever (i.e. identify with their heirs). This is a peculiar feature of the MCF criterion that I would argue makes it particularly difficult to implement in practice.

Third, the SOC criterion implies a social discount rate that is invariant to the model. The appropriate discount rate for evaluating costs and benefits (including any indirect revenue effects) is always and everywhere the rate of return foregone in the private sector. While indirect revenue effects are zero only if the project's benefits are equivalent to income, this seems like a more reasonable benchmark for most projects than assuming that benefits have no impact whatsoever on private sector behaviour.[16]

Throughout I have assumed that the pre-tax rate of return is exogenous. The "weighted average" discount rate is therefore simply the pre-tax rate of return. The cut off rate of return for any worthwhile project is therefore unique and common to all projects. In particular, it is independent of the time duration of costs and benefits. In models where the pre-tax rate of return is endogenous, the SOC will be a weighted average of the pre-tax and after-tax rates of return, the latter representing the social rate of time preference. This raises the question as to whether the weights depend upon the duration of the project. An early

[16] If there are indirect revenue effects (because the project's benefits are not equivalent to income) they would differ depending upon whether the ILA model or the OLG model is judged to be the most appropriate specification. In this respect the SOC criterion is also model sensitive.

paper by Diamond (1968) and a subsequent paper by Dreze (1974) made this point, but in the context of a model with a finite number of periods. However, if the tax regime is given and time invariant and the economy is in a steady state the cut off rate of return for all worthwhile projects will be independent of the duration of the project. Thus what is needed for project evaluation are estimates of benefits measured by willingness to pay, estimates of direct project expenditures, and estimates of indirect revenue effects whenever the project's compensated impact on distorted markets differs from zero. The appropriate discount rate is the social opportunity cost of capital, and it is independent of the project.

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